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ENERGY CONSERVATION RELATIONS IN NEWTON & GENERALIZED SPECIAL RELATIVISTIC MECHANICS & FORCE

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ABSTRACT

In this work it was shown that the energy conservation in Newtonian and generalized special relativity are equivalent. It also shown that when the energy is conserved the definition of force in terms of momentum Leads to its definition in terms of potential in both Newton and generalized special relativistic mechanics.

Keywords: *Energy conservation, Newton, generalized special relativity, momentum, potential.*

I. INTRODUCTION

Newtonian mechanics is one of the basic mechanical Law which describes macroscopic objects with Law velocity [1]. High speed microscopic objects can be described by special relativity (SR) [2]. However SR energy relation dose not redact to the Newtonian one for Law speed. This is since it does not have a term representing potential energy [3, 4]. This motivates some anthers to generalize SR by the so-called generalizedSR (GSR). This GSR energy equation successfully reduced to Newtonian one in the Law speed limit [5, 6]. In also explains gravitational redshirt and gravity time diction [6, 7]. These successes need searching for energy conservation requires mentis which is done in section (2). It also requires redefinition of force which is done in section (3). Sections (4) and (5) are concerned with discussion and conclusions.

II. ENERGY CONSERVATION FOR GSR

To see how energy is conserved consider the generalized special relativistic (GSR) energy equation

$$E = \frac{m_0 c^2}{\sqrt{1 - 2\varphi - \frac{v^2}{c^2}}} \qquad (2.1)$$

Rearranging and multiplying by m

$$E = \frac{m_0 c^2}{\sqrt{\frac{mc^2 - 2\left(m\varphi + \frac{1}{2}mv^2\right)}{mc^2}}} \qquad (2.2)$$

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$$E = \frac{m_0 c^2}{\sqrt{\frac{mc^2 - 2(V+T)}{E}}} \qquad (2.3)$$

Multiply both sides by the square root and squarely yields

$$E^2 \left(\frac{E - 2(T+V)}{E} \right) = m_0 c^2 \qquad (2.4)$$

$$E^2 - 2(T+V)E = m_0^2 c^4 \qquad (2.5)$$

If the sum of kinetic and potential energy is constant, i.e.

$$T + V = C_0 = constant$$

In this case equation (2.5) becomes

$$E^2 - c_0 E = m_0^2 c^4 \qquad (2.7)$$

Using the relation

$$ax^2 + bx + C_0 = 0 \qquad (2.8)$$

$$a = 1 \qquad b = -c_0 \qquad c = -m_0^2 c^4$$

$$E = -\frac{b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$E = +\frac{c_0 \pm \sqrt{b^2 - 4ac}}{2} \qquad (2.9)$$

Thus E is constant when the sum of T and V is constant

If on contrary E is contestant, i.e.

$$E = E_0 = constant \qquad (2.10)$$

Thus from (2.5)

$$E_0^2 - 2(T+V)E_0 = m_0^2 c^4$$

$$T + V = \frac{E_0^2 - m_0^2 c^4}{2E_0} = c_0 = constant \qquad (2.11)$$

Thus when E is constant the sum of T and V is constant.

Thus when Newtonian energy

$$E_N = T + V \qquad (2.12)$$

Is conserved GSR energy is also conserve.

III. DEFINATION OF FORCE IN TERMS OF POTENTIAL

A Newton low the energy wave equation is takes the form

$$E = \frac{P^2}{2m} + V = constant \qquad (3.1)$$

Where the total energy is constant differencing the equation (3.1) w.r.t time given

$$\frac{dE}{dt} = \frac{2p}{2m} \frac{dp}{dt} + \frac{dV}{dt} = 0 \qquad (3.2)$$

Where the momentum is given by

$$P = mv \qquad (3.3)$$

Using relation (3.3) and substituting in equation (3. 2)

$$v \frac{dp}{dt} = -\frac{dV}{dt} \qquad (3.4)$$

And the force is

$$F = \frac{dp}{dt} \qquad (3.5)$$

Using relation in (3.4)

$$F = -\frac{1}{v} \frac{dV}{dt} = -\frac{dt}{dx} \frac{dV}{dt} \qquad (3.6)$$

Thus equation (3.6) becomes

$$F = -\frac{dV}{dx} \qquad (3.7)$$

This is the formal definition of F in terms of V for special relativity one has:

$$E^2 = C^2 P^2 + m_0^2 C^4 \qquad (3.8)$$

By differenting equation (3.8) w.r.t time gives

$$2E \frac{dE}{dt} = 2\, C^2\, P \frac{dP}{dt} + 0 \qquad (3.9)$$

The force is defined as

$$\mathrm{F} = \frac{dP}{dt} \qquad (3.10)$$

Substituting in (3.9) and dividing by $(2c^2 p)$ yields

$$\frac{2E}{2C^2 P} \frac{dE}{dt} = \mathrm{F} \qquad (3.11)$$

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Thus

$$\frac{E}{C^2P}\frac{dE}{dt}=F \qquad (3.12)$$

But

$$P=mv \qquad E=mC^2 \qquad (3.13)$$

Substituting in (3.12) gives

$$\frac{mC^2}{C^2(mv)}\frac{dE}{dt}=\frac{1}{v}\frac{dE}{dt}$$

$$\frac{dt}{dx}\frac{dE}{dt}=\frac{dE}{dx}=F \qquad (3.14)$$

Which relates force to energy but according to GSR the energy satisfies

$$E=\frac{m_0C^2}{\sqrt{g_{00}-\frac{v^2}{c^2}}} \qquad (3.15)$$

$$E^2=\frac{m_0^2\,c^4E^2}{g_{00}E^2-C^2P^2} \qquad (3.16)$$

Divides by E^2

$$1=\frac{m_0^2\,c^4}{g_{00}E^2-C^2P^2} \qquad (3.17)$$

$$g_{00}E^2-C^2P^2=m_0^2\,c^4 \qquad (3.18)$$

Thus

$$g_{00}E^2=C^2P^2+m_0^2\,c^4 \qquad (3.19)$$

Differentiating $w.r.t\ t$ given

$$E^2\frac{dg_{00}}{dt}+g_{00}2E\frac{dE}{dt}=2\,c^2\ P\frac{dp}{dt} \qquad (3.20)$$

Rearranging and using the formal definition of force

$$\frac{dP}{dt}=F \qquad (3.21)$$

Rearranging again by dividing by $2C^2P$, yields

$$\frac{E^2}{2\,C^2P}\frac{dg_{00}}{dt}+\frac{2Eg_{00}}{2\,c^2P}\frac{dE}{dt}=F \qquad (3.22)$$

From energy equation $E = mc^2$ substituting in (3.22) gives

$$\frac{m^2c^4}{2\,c^2mv}\frac{dg_{00}}{dt} + \frac{2\,mc^2}{2\,c^2mv}\,g_{00}\frac{dE}{dt} = F$$

Thus

$$\frac{mc^2dt}{2\,dx}\quad\frac{dg_{00}}{dt} + \frac{dt}{dx}\;g_{00}\frac{dE}{dt} = F \qquad (3.23)$$

Thus

$$F = \frac{dP}{dt} = \frac{E}{2}\quad\frac{dg_{00}}{dx} + \quad g_{00}\frac{dE}{dx} \qquad (3.24)$$

$$g_{00} = 1 + \frac{2\emptyset}{c^2} \qquad (3.25)$$

Divides by m

$$g_{00} = 1 + \frac{2m\emptyset}{mc^2} = \left(1 + \frac{2V}{E}\right) \qquad (3.26)$$

Substituting

$$g_{00} = \left(1 + \frac{2V}{E}\right) \qquad (3.27)$$

Inserting (3.27) in (3.24) yields

$$F = \frac{dP}{dt} = \frac{E}{2}\quad\frac{2d(VE^{-1})}{dx} + \quad\left(1 + \frac{2V}{E}\right)\frac{dE}{dx}$$

Thus

$$F = \frac{E}{2}(2)\,(E)^{-1}\frac{dV}{dx} - VEE^{-2}\frac{dE}{dx} + \frac{dE}{dx} + \frac{2V}{E}\frac{dE}{dx} \qquad (3.28)$$

Cancelling similar terms in equation (3.28) yields

$$F = \frac{dV}{dx} - \frac{V}{E}\frac{dE}{dx} + \frac{dE}{dx} + \frac{2V}{E}\frac{dE}{dx} \qquad (3.29)$$

Thus

$$F = \frac{dV}{dx} + \frac{V}{E}\frac{dE}{dx} + \frac{dE}{dx} \qquad (3.30)$$

When E is conserved

$$\frac{dE}{dx} = 0 \qquad (3.31)$$

When the potential is positive, i.e. repulsive

$$V \rightarrow -V \qquad (3.32)$$

This is since g_{00} is derived by assuming negative attractive potential. Thus using (3.31) and (3.33) in equation (3.30) yields

$$F = \frac{\partial V}{\partial x} \qquad (3.33)$$

This is the ordinary definition of energy.

IV. DISCUSSION

Section (2) shows very interesting results. It shows that the conservation of GSR energy takes place when Newtonian one is conserved as shown by equation (2.6) and (2.9). If GSR is assumed to be conserved as equation (2.10) reads this Leads to Newtonian energy conservation as equation (2.12) shows.

The conservation of Newton and GSR energy indicates that the definition of force in terms of momentum time change Leads to the formal definition of force in terms of spatial change of potential. The Newtonian energy conservation in equation (3.1) beside force definition in terms of P in equation (3.5) Leads to definition F in terms of V as equation (3.7) indicates. For SR the definition of F in terms P Leads to definition of F in terms of E, with E replacing $(-V)$ as shown above. However for GSR the definition of F in terms of P in (3.21), with the constraint of energy conservation in (3.31) leads to the formal definition of F in terms of V as equation (3.33).

V. CONCLUSION

The conservation of Newtonian energy and GSR one are equivalent. This conservation explains when the formal of momentum time change and in terms of potential spatial change coincide.

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